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# Green's-Function Theory of the Parallel-Pumping Magnon Instability*

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The $(n+1)$-time Green's-function method is used to study the instability of a ferromagnetic system excited by a parallel-pumping field.

The Green's-function method has been widely used to calculate linear transport coefficients in statistical mechanics,[1] mainly because of its advantage of dealing directly with measurable quantities, avoiding the need for calculating wave functions. This method was extended by Tanaka *et al.*[2] for use in nonlinear systems, with the introduction of $(n+1)$-time Green's functions. Recently, Madsen and Tanaka[3] applied the $(n+1)$-time Green's-function method to the problem of parametric excitation of magnon modes by a rf magnetic field perpendicular to the dc field. To test the theory they established a criterion for estimating the threshold field for instability, obtaining good agreement with other results. In the present communication the Green's-function method is applied to the case of magnon excitation by a parallel-pumping field,[4,5] and the Madsen-Tanaka criterion is used to calculate the critical field.

Consider a ferromagnetic system magnetized to saturation by a static field $H$ ($z$ direction) and excited by a small parallel field $H_1\cos\omega t$, turned on adiabatically. The Hamiltonian of the system can be written as $\mathcal{H}=\mathcal{H}_0+\mathcal{H}_1$, where

$$\mathcal{H}_0=\sum_k \hbar\omega_k c_k^\dagger c_k \,, \tag{1}$$

$$\mathcal{H}_1=-\sum_k \hbar H_1\cos\omega t\,(\rho_k c_k^\dagger c_{-k}^\dagger+\rho_k^* c_k c_{-k}) \equiv B\cos\omega t/2, \tag{2}$$

where $c_k^\dagger$ and $c_k$ are, respectively, the magnon creation and destruction operators, $\hbar\omega_k$ is the magnon energy, and $\rho_k$ is given by[6]

$$\rho_k=\pi\gamma^2 M\omega_k^{-1}\sin^2\theta_k e^{-i2\phi_k}\,, \tag{3}$$

where $\gamma$ is the gyromagnetic ratio, $M$ is the saturation magnetization, and $\theta_k$ and $\phi_k$ are the polar angles that characterize the direction of the wave vector $\vec{k}$. The perturbation term (2) has been previously discussed by Schlömann[6] who has also shown that the rf part of the $z$ component of the magnetization vector can be expressed as

$$A=\sum_k \hbar\rho_k c_k^\dagger c_{-k}^\dagger+\text{c.c.}=\sum_k A_k\,. \tag{4}$$

The $(n+1)$-time Green's function for $A$ can be defined as

$$G_{n+1}(A(\tau_n+\cdots+\tau_1);B(\tau_{n-1}+\cdots+\tau_1);\ldots;B(\tau_1);B)$$
$$\equiv(-i)^n\theta(\tau_n)\theta(\tau_{n-1})\cdots\theta(\tau_1)\times\langle[[\cdots[A(\tau_n+\cdots+\tau_1),B(\tau_{n-1}+\cdots+\tau_1)]\cdots,B(\tau_1)],B]\rangle\,, \tag{5}$$

where $A(\tau)$ and $B(\tau)$ are the interaction representation of $A$ and $B$. For a harmonic perturbation as in (2) the fundamental frequency part of the thermodynamic average of $A$ is[3]

$$\langle A\rangle_{\text{av}}=e^{-i\omega t}\{(-2\pi)G_2(A;\omega)+(-2\pi)^3[G_4(A;\omega,0,-\omega)+G_4(A;\omega,0,\omega)+G_4(A;\omega,2\omega,\omega)]\}\,, \tag{6}$$

where $G_{n+1}(A;\omega_n,\omega_{n-1},\ldots,\omega_1)$ is the Fourier transform of (5). Making use of the boson commutation relations, the equations of motion[3] for $G_2$ and $G_4$ can be readily solved to give

$$G_2(A_k;\omega)\simeq(2\pi)^{-1}H_1(\hbar|\rho_k|)^2(1+2n_k^0)(\omega-2\omega_k+2i\eta_k)^{-1},$$
$$G_4(A_k;\omega,0,\omega)\simeq-(2\pi)^{-2}(H_1\hbar|\rho_k|)^2 i\eta_k^{-1}\times(\omega-2\omega_k+2i\eta_k)^{-1}G_2(A_k;\omega)\,,$$
$$G_4(A_k;\omega,0,-\omega)\simeq-(2\pi)^{-2}(H_1\hbar|\rho_k|)^2 i\eta_k^{-1}\times(-\omega+2\omega_k+2i\eta_k)^{-1}G_2(A_k;\omega)\,, \tag{7}$$
$$G_4(A_k;\omega,2\omega,\omega)\simeq 0\,,$$

where $n_k^0$ is the $k$-mode thermal equilibrium occupation number. The approximations were made on account of the denominators being very small at $\omega\simeq 2\omega_k$. The damping constant was introduced by assigning an imaginary part to the frequency $\omega_k$. As evidenced in Eqs. (3) and (7) the magnon pairs propagating perpendicularly to the static field ($\theta_k=\pi/2$) and satisfying the energy condition $\omega=2\omega_k$ are the most strongly excited. The magnetic susceptibility obtained from (7) corresponds to the result of Joseph and Schlömann[7] expanded to second order in the field amplitude. Following Madsen and Tanaka[3] we establish the condition for the onset of the instability process

$$|G_2(A;\omega)|=(2\pi)^2\hbar^{-2}|G_4(A_k;\omega,0,\omega)+G_4(A_k;\omega,0,-\omega)+G_4(A_k;\omega,2\omega,\omega)|\,, \tag{8}$$

which gives the critical field amplitude previously obtained[4,5]

$$H_{1c} = |\rho_k|^{-1}[(\tfrac{1}{2}\omega - \omega_k)^2 + \eta_k^2]^{1/2} . \qquad (9)$$

In the case of Madsen and Tanaka[3] there is not a perfect agreement with the result of other theories, whereas in our case Eq. (9) is exact. The instability criterion (8) indicates the intensity of the excitation for which the Green's-function series does not converge. This criterion should also be applicable to determine the threshold condition in other nonlinear effects, such as the stimulated Raman or Brillouin scattering.

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[1]S. V. Tyablikov, *Methods in the Quantum Theory of Magnetism* (Plenum, New York, 1967).
[2]T. Tanaka, K. Moorjani, and T. Morita, Phys. Rev. 155, 388 (1967).
[3]E. Madsen and T. Tanaka, Phys. Rev. 184, 527 (1969).
[4]F. R. Morgenthaler, J. Appl. Phys. Suppl. 31, 95 (1960).
[5]E. Schlömann, J. J. Green, and U. Milano, J. Appl. Phys. Suppl. 31, 386 (1960).
[6]E. Schlömann, J. Appl. Phys. 34, 1998 (1963).
[7]R. I. Joseph and E. Schlömann, J. Appl. Phys. 38, 1915 (1967).

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# Note on Gadzuk's Theory of Field-Induced Tunneling

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Recently, Gadzuk[1,2] presented theoretical studies concerning band-structure effects on field-induced tunneling from metals and resonant tunneling through atoms absorbed on metal surfaces. The calculation of the matrix elements necessary for comparison with experiment in these papers appears to be seriously in error and, at least in the second paper, to cast doubts on the usefulness of these calculations for comparison with the experimental work of Plummer and Young.[3] The basic error is the statement $j_l(ikr) = i^l j_l(kr)$. That this is incorrect can be seen clearly by considering the case $l=0$ where $j_0(x) = x^{-1}\sin x$. The matrix elements are easily recalculated by replacing $j_l(x)$ by $(\pi/2x)^{1/2} I_{l+1/2}(x)$ wherever it occurs. Thus, for example, Eqs. (35a) and (35b) of Ref. 2 should read

$$I_1 = \frac{\Gamma(7)}{k^8(\beta_s^2 - 1)^7}(7\beta_s^6 + 35\beta_s^4 + 21\beta_s^2 + 1) ,$$

$$I_2 = \frac{3\Gamma(8)}{k^{10}(\beta_s + 1)^8} + \frac{\Gamma(9)}{2k^{10}}[(\beta_s - 1)^{-9} - (\beta_s + 1)^{-9}] .$$

Hence, for $k \approx 1.2$ Å$^{-1}$, $a_s \approx 1.5$ Å$^{-1}$, $I_1(\mathrm{corr})/I_1(35a) \simeq -25$, $I_2(\mathrm{corr})/I_2(35b) \simeq 30\,000$.

[1]J. W. Gadzuk, Phys. Rev. 182, 416 (1969).
[2]J. W. Gadzuk, Phys. Rev. B 1, 2110 (1970).
[3]E. W. Plummer and R. D. Young, Phys. Rev. B 1, 2088 (1970).

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# Reply to Comments on a Theory of Field-Induced Tunneling

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In the preceding paper,[1] Glasser has pointed out an oversight in some manipulative details leading to approximate expressions in this writer's theoretical studies of field-induced tunneling.[2,3] However, the mishap is not as disastrous as it might first appear. In all cases in which the tunneling matrix elements were calculated, the physically significant results were presented in the form of ratios of ma-